Lifshitz/Schrödinger Dp-branes and dynamical exponents

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Abstract

We extend our earlier study of special double limits of 'boosted' AdS_5 black hole solutions to include all black Dp-branes of type II strings. We find that Lifshitz solutions can be obtained in generality, with varied dynamical exponents, by employing these limits. We then study such double limits for 'boosted' Dp-brane bubble solutions and find that the resulting non-relativistic solutions instead describe Schrödinger like spacetimes, having varied dynamical exponents. We get a simple map between these Lifshitz & Schrödinger solutions and a relationship between two types of dynamical exponents. We also discuss about the singularities of the Lifshitz solutions and an intriguing thermodynamic duality.

1 Introduction

The holographic studies involving non-relativistic string theory backgrounds have received good attention recently, both from the point of view of uncovering AdS/CFT holographic principle, as well as to understand some quantum critical phenomena in strongly correlated condensed matter systems. The condensed matter systems by nature are usually nonrelativistic but in some systems near some critical point the system may appear to show exotic scaling behaviour. These could be systems, for example atomic gases at ultra low temperatures, or fermions at unitarity [1, 2]. Such systems are known to exhibit Schrödinger or Lifshitz like behaviour.

There have been namely two types of non-relativistic (Galilean) string backgrounds, exhibiting asymmetric space and time scaling symmetry, which have been a subject of several studies recently [1]-[30]. The ones which exhibit Schrödinger symmetries [1, 2] are described as

$$ds^2_{Sch} = \left(-\frac{(dx^+)^2}{z^{2a}} - \frac{dx^+ dx^-}{z^2} + \frac{dx_i^2}{z^2} \right) + \frac{dz^2}{z^2} \tag{1}$$

and those having Lifshitz type symmetries [4] are like

$$ds^2_{Lif} = \left(-\frac{dt^2}{z^{2a}} + \frac{dx_i^2}{z^2} \right) + \frac{dz^2}{z^2}. \tag{2}$$

In both these cases, x^i's are flat spatial coordinates while z is known as the holographic coordinate (which is a measure of the energy scale of the boundary theory). The constant parameter a is called the dynamical exponent of time. These bulk geometries are suitable to describe an asymmetric scaling quantum phenomena at some critical point in a nonrelativistic CFT, which resides on the boundary of such spacetimes. The Schrödinger spacetimes can be obtained as solutions in string theory [5, 6]. Not only this, the nonrelativistic spaces can also be obtained as solutions of 'massive' type IIA string theory [15]. More recently, the Lifshitz-like spaces have been embedded variously in ordinary string theory as demonstrated by [18, 19, 21]. These explicit examples imply that a wide class of non-relativistic solutions can be constructed in string theory and hopefully some of these could be engineered to explain some strongly coupled phenomena in respective boundary theories. In this work we shall show that a class of these solutions indeed can be obtained consistently as limiting cases of well studied AdS Dp brane solutions.

We wish to extend our last study of D3-branes where vanishing horizon limits of 'boosted' $AdS_5 \times S^5$ black holes give rise to the Lifshitz solutions. These $a = 3$ Lifshitz solutions and similar solutions in M-theory with fractional dynamical exponents were studied in [21, 22]. Repeating the same methods for all Dp-branes here, we obtain a wide class of Lifshitz and Schrödinger type solutions describing nonrelativistic systems with varied dynamical exponents. The paper is organized as

follows. In section-II we introduce 'boosted' black Dp-brane solutions. These solutions have two parameters namely, the horizon radius and the boost velocity, given by r_0 and γ. We discuss a special double limit in which the black hole horizon is allowed to shrink while the boost is simultaneously taken to be very large, but keeping their product fixed. The new zero temperature geometries thus obtained describe Lifshitz type dynamics with varied dynamical exponents as $a_{Lif} = 2(p-6)/(p-5)$. In section-III we study similar limits of the 'boosted' bubble p-brane solutions. The resulting solutions are instead of Schrödinger type solutions with dynamical exponents $a_{Sch} = 2/(p-5)$. We do however find an interesting map between these Lifshitz and Schrödinger solutions under which dynamical exponents get mapped into each other. The section-IV we discuss thermodynamical aspects of Lifshitz solutions and singularities. We also report on an intriguing thermodynamical duality. The conclusion is given in section-V.

2 Double limits of non-BPS Dp-brane geometries

We start with the black Dp-brane solutions which are asymptotically AdS solutions and describe temperature phases of the boundary CFTs. [1] (We shall take $\alpha' = 1$). These AdS black hole solutions are

$$ds^2_{BH} = R_p^2 r^{\frac{p-3}{2}} \left[r^{5-p}(-f dt^2 + d\vec{x}^2_{(p)}) + \frac{dr^2}{fr^2} + d\Omega^2_{(8-p)} \right],$$

$$e^\phi = (2\pi)^{2-p} g_{YM}^2 \left(\frac{R_p^4}{r^{7-p}} \right)^{\frac{3-p}{4}} \sim \frac{\lambda_{eff}^{\frac{7-p}{4}}}{N} \ , \quad \text{for } 0 \le p \le 6 \tag{3}$$

along with electric flux for $(p+2)$-form field strength which couples to electrically charged p-branes $(p < 3)$ as

$$F_{p+2} \simeq (7-p)(R_p)^{2p-2} r^{6-p} dr \wedge dt \wedge [dx_{(p)}] \ . \tag{4}$$

For the magnetically charged solutions $(p > 3)$ one instead takes

$$F_{8-p} \simeq (7-p)(R_p)^4 \omega_{(8-p)}, \tag{5}$$

with $\omega_{(8-p)}$ being the volume form of a unit size rigid sphere, S^{8-p}. Especially for the D3-branes field strength $F_{(5)}$ should be taken self-dual. In the metric $x_1, \cdots, x_p$ are flat spatial coordinates along the Dp-brane world-volume, N is the number of the branes, whereas $d\Omega^2_{(8-p)}$ represents the line element of unit $(8-p)$-dimensional sphere, and $(R_p)^2 \equiv g_{YM}\sqrt{d_p N}$ represents radius square of the sphere. [2] In the

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[1] We shall be generally using the common phrases 'AdS-black hole' and 'AdS-Bubble' in this work, but the reader is alerted to keep in mind that the near horizon geometries such as (3) are usually of the type $\Omega^2(x)(ds^2[AdS_{p+2}] + ds^2[S^{8-p}])$ and there is a running dilaton field, except for $p = 3$ case where conformal factor is constant. Nevertheless there is a well defined $(p+1)$-dimensional boundary CFT description on the boundary of AdS_{p+2} subspace. We have also suitably rescaled t, x^i in Eqs.(3) and (4) so as to absorve some R_p factors.

[2] Here d_p comprises suitable combinatoric factor of 2's and π's.

function $f(r) = 1 - r_0^{7-p}/r^{7-p}$, $r = r_0$ is the size of black hole horizon. Asymptotically as $r \to \infty$ the spacetime metric becomes conformal to spacetime which is a direct product of an AdS_{p+2} and S^{8-p}. The boundary conformal field theory will have finite temperature where t could be taken imaginary ($i\tau$) and suitably periodic, $\tau \sim \tau + \frac{4\pi}{(7-p)} r_0^{\frac{p-5}{2}}$. The effective 't Hooft coupling of the boundary SYM theory is given as

$$\lambda_{eff} \sim \lambda_0 r^{p-3},$$

while $\lambda_0 = g_{YM}^2 N$ being the bare 't Hooft coupling constant. [3] For our purpose here we have instead taken noncompact Lorentzian time. We shall restrict to the branes with $1 \leq p \leq 6$. For the D0-branes, as they are point like we shall not study them here.

We make the following boost transformation involving one of the brane direction, say y,

$$dt \to \gamma dt + v\gamma dy, \quad dy \to \gamma dy + v\gamma dt \tag{6}$$

where velocity $0 \leq v < 1$ ($c = 1$) and the boost parameter $\gamma = 1/\sqrt{1-v^2}$. Since there is no Lorentz symmetry along all brane directions, particularly so involving the y direction, so we get a new boosted BH geometry

$$ds^2 = R_p^2 r^{\frac{p-3}{2}} \left[r^{5-p} \left((1 + \frac{r_0^{7-p} v^2 \gamma^2}{r^{7-p}}) dy^2 - (1 - \frac{r_0^{7-p} \gamma^2}{r^{7-p}}) dt^2 + \frac{2v\gamma^2 r_0^{7-p}}{r^{7-p}} dt dy + d\vec{x}_{(p-1)}^2 \right) \right.$$
$$\left. + \frac{dr^2}{fr^2} + d\Omega_{(8-p)}^2 \right]$$
$$e^\phi = (2\pi)^{2-p} g_{YM}^2 \left(\frac{r^{7-p}}{R_p^4} \right)^{\frac{p-3}{4}}, \quad \text{for } 1 \leq p \leq 6 \tag{7}$$

while rest of the background fields remain unchanged under the boost, so we shall not writing them again here after, but it should be understood that all branes are charged so we need appropriate flux. Here we would like to note that the periodicity of Euclidean t in the boosted geometry gets modified and it is now $\frac{4\pi\gamma}{(7-p)} r_0^{\frac{p-5}{2}}$. Correspondingly black hole horizon temperature is

$$T = \frac{(7-p)r_0^{\frac{5-p}{2}}}{4\pi\gamma}. \tag{8}$$

The simultaneous $r_0 \to 0$, $\gamma \to \infty$ double limits:

While setting $r_0 = 0$ in (7) will give rise to usual AdS brane solutions, we instead like to consider a double limit in which r_0 is allowed to vanish while boost is simultaneously taken to be infinity, such that

$$r_0 \to 0, \quad \gamma \to \infty, \quad r_0^{7-p} \gamma^2 = \beta^2 = \text{fixed}. \tag{9}$$

In which case we have

$$(1 + f) \sim 2 - O(r_0^{7-p}), \quad (1 - f)\gamma^2 \to \frac{\beta^2}{r^{7-p}} \tag{10}$$

Introducing light-cone coordinates through $x^\pm = (t \pm y)$, the solution (7) reduces to

$$ds^2_{Lif} = R_p^2 r^{\frac{p-3}{2}} \left[r^{5-p}(\frac{\beta^2}{r^{7-p}}(dx^-)^2 - dx^+ dx^- + d\vec{x}^2_{(p-1)}) + \frac{dr^2}{r^2} + d\Omega^2_{(8-p)} \right]$$

$$e^\phi = (2\pi)^{2-p} g^2_{YM} (\frac{r^{7-p}}{R_p^4})^{\frac{p-3}{4}}, \tag{11}$$

with suitable

$$F_{p+2} = (7 - p) R_p^{2p-2} r^{6-p} dr \wedge dx^+ \wedge dx^- \wedge [dx_{(p-1)}]$$

for the electric type branes $(p < 3)$ and

$$F_{8-p} = (7 - p) R_p^4 \omega_{8-p}$$

for magnetic type branes $(p > 3)$. For D3-branes instead we shall have $F_5 = 4(1 + \star)\omega_5$ which is self-dual.

The temperature vanishes as r_0^{6-p} for $p < 6$. Thus our limits imply vanishing temperature as $r_0 \to 0$ for $p < 6$. Especially for $p = 6$, the temperature can be arbitrary. We immediately notice that light-cone time is null, i.e. $g_{++} = 0$, in these solutions. But a finite g_{++} emerges once we compactify along x^-. (Since g_{--} component is finite we can compactify along x^- lightcone coordinate. This would give rise to a Lifshitz like geometry in lower dimensions [21]. Also see [19] for $a = 2$ Lifshitz solutions where similar situations arise.) Nevertheless the noncompact solutions (11) make complete solutions of type IIA/B string theory depending upon whether p is even/odd. We can see that the g_{--} component of the metric is subdominant compared to g_{+-} and g_{ii} when $r \to \infty$. Thus asymptotically the metric (7) will become

$$ds^2_{Lif} \sim R_p^2 r^{\frac{p-3}{2}} \left(r^{5-p}[-dx^+ dx^- + d\vec{x}^2_{(p-1)}] + \frac{dr^2}{r^2} + d\Omega^2_{(8-p)} \right)$$

$$e^\phi = (2\pi)^{2-p} g^2_{YM} (\frac{r^{7-p}}{R_p^4})^{\frac{p-3}{4}} \tag{12}$$

along with a suitable $(p + 2)$-form flux. These precisely are the near horizon (relativistic) geometries representing multiple p-brane systems. Thus we see that, for all $1 \leq p \leq 6$, the nonrelativistic deformations (β-terms in the metric) become prominent only in the IR region. However this metric deformation will require us to switch on appropriate operators in the boundary field theory. However, these operator deformations will disappear in the UV regime of corresponding CFT. Also due to these

nonrelativistic deformations in the IR regime, the scaling behaviours of space and time changes drastically. The time and space will now scale asymmetrically and we get Lifshitz like behaviour.

Scaling symmetry:

Let us redefine the radial coordinate as

$$r^{p-5} = z^2. \tag{13}$$

With z as holographic coordinate and some scalings the solutions can be written as (for $p \neq 5$)

$$ds^2_{Lif} = R_p^2 z^{\frac{p-3}{p-5}} \left[\left(\frac{2}{5-p} \right)^2 \left(\frac{\beta^2}{z^{4/(p-5)}} (dx^-)^2 + \frac{-dx^+ dx^- + d\vec{x}^2_{(p-1)}}{z^2} + \frac{dz^2}{z^2} \right) + d\Omega^2_{(8-p)} \right]$$
$$e^\phi = (2\pi)^{2-p} g^2_{YM} R_p^{3-p} (z^{\frac{2(7-p)}{p-5}})^{\frac{p-3}{4}} \tag{14}$$

with the $(p+2)$-form flux. We find that under asymmetric scalings (dilatation) of the coordinates

$$z \to \xi z, \quad x^- \to \xi^{2-a} x^-, \quad x^+ \to \xi^a x^+, \quad \vec{x} \to \xi \vec{x} \tag{15}$$

with dynamical exponent $a = \frac{2(p-6)}{p-5}$, the dilaton and the metric in (14) conformally rescale as

$$g_{MN} \to \xi^{\frac{p-3}{p-5}} g_{MN}, \quad e^\phi \to \xi^{\frac{(7-p)(p-3)}{2(p-5)}} e^\phi \tag{16}$$

This is nothing but the standard Weyl rescaling behaviour of near horizon Dp-branes AdS solutions [31]. The Weyl scaling of the bulk metric and e^ϕ is indicative of the fact that the boundary non-relativistic CFT (NRCFT) is not a conformal theory instead it has got a running effective coupling (λ_{eff}). But what we find most surprising is the fact that the RG flow is still of the standard type inspite of the asymmetric scalings of the coordinates. Only for D3-branes we have a scaling symmetry which involves no Weyl rescaling of the bulk metric. For D3-brane case, discussed earlier in [21], one finds a Lifshitz spacetime

$$ds^2_{D3} = R_3^2 \left[\left(\frac{\beta^2}{z^{-2}} (dx^-)^2 + \frac{-dx^+ dx^- + d\vec{x}^2_{(2)}}{z^2} \right) + \frac{dz^2}{z^2} + d\Omega^2_{(5)} \right]$$
$$e^\phi = (2\pi)^{-1} g^2_{YM} , \quad F_{(5)} = 4R_3^4 (1 + \star) \omega_5 \tag{17}$$

where the dynamical exponent is $a = 3$. These 5-dimensional spaces were previously also known as Kaigorodov spacetimes in literature or as 'Einstein spaces of maximum mobility' [33, 34]. The dual field theory thus involves an infinitely boosted CFT. If we mainly focus on the coordinate patch $(x^+, x^-, \vec{x})$ at the boundary where the CFT lives, we can see that the spacetime indeed possess asymmetric scaling

symmetry. Since x^- coordinate is not null, we can also think of compactifying along this direction. In [21] it was shown that the lower dimensional solution resembles a Lifshitz geometry. To see this we could rewrite the metric of Eq.(17) in a diagonal basis as

$$ds^2_{D3} \;=\; R_3^2 \left([-\frac{(dx^+)^2}{4\beta^2 z^6} + \frac{dx_1^2 + dx_2^2 + dz^2}{z^2}]_{Lif_4} + \beta^2 z^2 (dx^- - \frac{dx^+}{2\beta^2 z^4})^2 + d\Omega_5^2 \right)$$

(18)

From metric Eq.(18) it is obvious that the nonrelativistic geometry indeed represents a system of Kaluza-Klein particles (gravi-photons) in lower dimensions. That is upon compactification, we will simply be dealing with, a dilaton and graviphotons in a 4-dimensional Lifshitz universe $\left(-\frac{(dx^+)^2}{4\beta^2 z^6} + \frac{dx_1^2 + dx_2^2 + dz^2}{z^2}\right)$. This story will repeat itself for all p-branes as and when we compactify the x^- direction.

Note that once x^- is compactified, $x^- \sim x^- + 2\pi r^-$, the Lifshitz geometry (17) acquires a complete distinct notion. They can provide a valid holographic description but only in a definite z range. For example, solutions (18) cannot be trusted near the AdS boundary (in UV) because the physical size of x^- circle

$$\frac{R^-_{phys}}{l_s} = \frac{R_3}{l_s} \beta r^- z$$

(19)

becomes sub-stringy as $z \to 0$. This would necessitate higher derivative string (world-sheet) corrections to the solutions. Alternatively, as suggested in [6], when such a situation arises, it will also be appropriate to go over to a T-dual type II string solution where the T-dualised x^- circle can have a finite radius.

In addition to the scaling properties in Eqs.(15),(16) as described earlier, the Lifshitz solutions (11) or (14) also have invariances under the translations

$$x^+ \to x^+ + b^+, \quad x^- \to x^- + b^-, \quad x^i \to x^i + b^i$$

(20)

and under the rotations of x^i coordinates. However, due to nontrivial g_{--} components these spaces (17) do not have any explicit invariance under the Galilean boost

$$x^+ \to x^+, \quad x^- \to x^- - 2\vec{v}.\vec{x} + v^2 x^+, \quad \vec{x} \to \vec{x} - \vec{v}x^+ ,$$

(21)

$\vec{v}$ is constant velocity. However there exists an unusual symmetry under space-like shifts of the time (x^+)

$$x^- \to x^-, \quad x^+ \to x^+ - 2\vec{v}.\vec{x} + v^2 x^-, \quad \vec{x} \to \vec{x} - \vec{v}x^-.$$

(22)

The latter shift symmetry will be absent when x^- coordinate is compactified. Thus our noncompact solutions (14) represent Lifshitz geometry when compactified along x^-, in which the time scales asymmetrically with dynamical exponent

$$a_{Lif} = 2\frac{p-6}{p-5}.$$

(23)

Note, however, no extra matter fields are present in the backgrounds (11) except dilaton and $(p+2)$-form fluxes.

Let us now find the supersymmetries preserved by the Lifshitz solutions (11). Since all these solutions are in the same class, let us pick up the simplest case of D3-branes. We have checked that for $p = 3$ Lifshitz backgrounds (11) all superconformal Killing spinors identically vanish, while remaining sixteen Poincaré Killing spinors have to satisfy an additional condition $\Gamma^+ \epsilon = 0$. Thus only 8 Poincaré supersymmetries survive for the Lifshitz background, so they are only 1/4-BPS solutions, see also [21]. The same would be true for the case of p other than 3.

Separately, for D5-branes we get the solution as

$$ds^2_{D5} = R_5^2 \frac{1}{z}\Big[(\beta^2 z^2 (dx^-)^2 - dx^+ dx^- + d\vec{x}^2_{(4)}) + \frac{dz^2}{z^2} + d\Omega^2_{(3)}\Big]$$

$$e^\phi = (2\pi)^{-3}\frac{g^2_{YM}}{R_5^2 z}, \quad F_{(3)} = 2R_5^4 \omega_{(3)} \tag{24}$$

where Lifshitz scaling is $z \to \xi z$, $x^+ \to \xi x^+$, $x^- \to \xi^{-1} x^-$, $x^i \to x^i$ along with the Weyl rescalings. Note x^i's do not need to rescale in these 5-brane solutions and they have dynamical exponent $a = 1$. So they appear to remain relativistic branes, but they are not due to nontrivial g_{--} metric component.

For D6-branes from (14) we have

$$ds^2_{D6} = R_6^2 z^3 \Big[4(\frac{\beta^2}{z^4}(dx^-)^2 + \frac{-dx^+ dx^- + d\vec{x}^2_{(5)}}{z^2} + \frac{dz^2}{z^2}) + d\Omega^2_{(2)}\Big]$$

$$e^\phi = (2\pi)^{-4} g^2_{YM}(\frac{z}{R_6^2})^{\frac{3}{2}} \tag{25}$$

where scaling invariance is $z \to \xi z$, $x^+ \to x^+$, $x^- \to \xi^2 x^-$, $x^i \to \xi x^i$ along with Weyl rescaling. Note x^+ does not need to rescale in these 6-brane Lifshitz solutions, so they have dynamical exponent of time as $a = 0$. So for these solutions, their Euclidean counter part can have x^+ with arbitrary periodicity, which means an arbitrary unfixed temperature.

3 Double limits of AdS-Bubble geometries and Schrödinger spacetime

Here we start with the 'bubble' spacetimes which are asymptotically AdS solutions of type II string theory. The AdS-bubble solutions are well known to describe the low temperature phases in their respective holographic SYM theories. These bubble geometries are

$$ds^2_{Bubble} = R_p^2 r^{\frac{p-3}{2}}\Big[r^{5-p}(-dt^2 + f dy^2 + d\vec{x}^2_{(p-1)}) + \frac{dr^2}{fr^2} + d\Omega^2_{(8-p)}\Big],$$

$$e^\phi = (2\pi)^{2-p} g^2_{YM}(\frac{r^{7-p}}{R_p^4})^{\frac{p-3}{4}} \tag{26}$$

with appropriate flux for F_{p+2} form field strength. We shall consider again $1 \leq p \leq 6$. We note here that the above bubble p-brane geometries can be obtained by performing a double Wick rotation

$$t \to iy, \quad y \to it \tag{27}$$

of the black hole solutions given in (3). But they make two physically distinct solutions, one involves black holes and the others do not. In the above $f(r) = (1 - r_b^{7-p}/r^{7-p})$ with r range being $r_b \leq r \leq \infty$. Since the radial coordinate is restricted in the IR region, they describe what is commonly known as 'bubble geometry', inside of the bubble is empty. The coordinate y has to have correct periodicity, $y \sim y + \frac{4\pi}{(7-p)} r_b^{\frac{p-5}{2}}$, in order that metric avoids the singularity at $r = r_b$ and assumes a sigar like spatial geometry. Asymptotically bubble spacetimes become conformal to a geometry which is a product of an AdS spacetime with a compact spatial coordinate and a round sphere, and having a running dilaton field, except for the case of $p = 3$ case where dilaton becomes constant. The boundary is at $r \to \infty$. Since r is the holographic (energy) coordinate, the boundary conformal field theory has an effective IR cut-off scale as $r = r_b$. The field theory can have finite small temperature if the Euclidean t is taken to be periodic. The period of Euclidean time can be arbitrary in these solutions but it is usually fixed by the temperature of the boundary CFT. Here we have taken Lorentzian time for the purpose of this work.

We employ the following boost transformation along compact y coordinate

$$dt \to \gamma dt + v\gamma dy, \quad dy \to \gamma dy + v\gamma dt \tag{28}$$

with boost parameter $\gamma = 1/\sqrt{1 - v^2}$. Since there is no Lorentz symmetry involving y direction, we get a boosted bubble metric

$$ds^2 = R_p^2 r^{\frac{p-3}{2}} \left[r^{5-p} \left(-(1 + \frac{r_b^{7-p} v^2 \gamma^2}{r^{7-p}}) dt^2 + (1 - \frac{r_b^{7-p} \gamma^2}{r^{7-p}}) dy^2 + \frac{2v\gamma^2 r_b^{7-p}}{r^{7-p}} dt dy + d\vec{x}_{(p-1)}^2 \right) \right.$$
$$\left. + \frac{dr^2}{fr^2} + d\Omega_{(8-p)}^2 \right] \tag{29}$$

while other background fields remain unchanged under the boost. Here we would like to note that the y-periodicity in the boosted geometry gets modified to $y \sim y + \frac{4\pi\gamma}{(7-p)} r_b^{\frac{p-5}{2}}$,

The simultaneous $r_b \to 0$, $\gamma \to \infty$ limit:

In order to find nonrelativistic brane solutions we consider double limits in which r_b is allowed to vanish while boost is simultaneously taken to be large, such that

$$r_b \to 0, \quad \gamma \to \infty, \quad r_b^{7-p} \gamma^2 = \beta^2 = \text{fixed}. \tag{30}$$

9

Note that while we take these double limits y coordinate indeed gets decompactified ($p < 6$). In lightcone coordinates the solution (29) reduces to

$$ds^2_{Sch} = R_p^2 r^{\frac{p-3}{2}} \left[r^{5-p}(-\frac{\beta^2}{r^{7-p}}(dx^+)^2 - dx^+ dx^- + d\vec{x}^2_{(p-1)}) + \frac{dr^2}{r^2} + d\Omega^2_{(8-p)} \right]$$

$$e^\phi = (2\pi)^{2-p} g_{YM}^2 (\frac{R_p^4}{r^{7-p}})^{\frac{3-p}{4}} \tag{31}$$

with appropriate $(p+2)$-form flux. We can notice that x^- is null in these Schrödinger type solutions. Which is juxtapose of the Lifshitz like solutions obtained earlier from AdS black holes. Although, x^- is an isometry direction, but as it remains a null direction, it will not be possible to compactify along this lightcone coordinate. Indeed the situation is much like for Schrödinger solutions with dynamical exponent 2, see Maldacena et.al [6]. Nevertheless the Schrödinger solutions (31) are complete solutions of type II string theory. Asymptotically in the region where $r^{7-p} \gg \beta^2$ the metrics precisely become AdS geometries representing multiple p-branes. Thus for all $p \le 6$ the nonrelativistic (Schroedinger type) deformations (namely g_{++} components) become prominent only in the IR region. While all NR effects disappear in the UV regime.

Asymmetric scaling symmetry:

Let us introduce the holographic z-coordinate again. With z coordinate the solutions become (for $p \ne 5$)

$$ds^2_{Sch} = R_p^2 z^{\frac{p-3}{p-5}} \left[\frac{4}{(5-p)^2} [-\frac{\beta^2}{z^{\frac{4}{p-5}}}(dx^+)^2 + \frac{-dx^+ dx^- + d\vec{x}^2_{(p-1)}}{z^2} + \frac{dz^2}{z^2}] + d\Omega^2_{(8-p)} \right]$$

$$e^\phi = (2\pi)^{2-p} g_{YM}^2 (R_p^4 z^{\frac{2(p-7)}{p-5}})^{\frac{3-p}{4}} \tag{32}$$

with $(p+2)$ form flux. Thus we see that there is an asymmetric scaling invariance involving the coordinates

$$z \to \xi z, \quad x^- \to \xi^{2-a} x^-, \quad x^+ \to \xi^a x^+, \quad \vec{x}_{p-1} \to \xi \vec{x}_{p-1} \tag{33}$$

with dynamical exponent

$$a_{Sch} = \frac{2}{p-5}, \tag{34}$$

where dilaton and the metric conformally scale as

$$g_{MN} \to \xi^{\frac{p-3}{p-5}} g_{MN}, \quad e^\phi \to \xi^{\frac{(7-p)(p-3)}{2(p-5)}} e^\phi \tag{35}$$

It is nothing but precisely the usual Weyl scaling behaviour of the Dp-branes near horizon AdS geometries. If we focus only on the coordinate patch $(x^+, x^-, \vec{x})$ where the CFT lives, we see that the CFT will exhibit Schrödinger symmetry but devoid of special conformal symmetry. The solutions (32) are also invariant under translations, rotations and Galilean boosts Eqs. (20) and (22).

Especially for D3-branes case we find

$$ds_{D3}^2 = R_3^2\bigg[-\beta^2 z^2 (dx^+)^2 + \frac{-dx^+ dx^- + d\vec{x}_{(2)}^2}{z^2} + \frac{dz^2}{z^2} + d\Omega_{(5)}^2 \bigg]$$
$$e^\phi = (2\pi)^{-1} g_{YM}^2, \quad F_{(5)} = 4R_3^4(1 + \star)\omega_5 \qquad (36)$$

which is a Schrödinger spacetime but with the dynamical exponent as $a = -1$. Let us note that Schrödinger spacetimes with dynamical exponent $a = 2$ only admit special conformal symmetry [1, 2].

The Schrödinger spacetime with $a = -1$ can also be called a Kaigorodov space. The Kaigorodov spaces are known as Einstein spacetimes of maximum mobility [33]. Corresponding boundary dual theories have been described as CFTs in infinite momentum frame [34]. So it is not surprising that we obtained our solutions from boosted 'black' and 'bubble' p-brane solutions under the limits involving infinite boosts. For example, the Ricci tensor for the 5D Kaigorodov space is simply given by $R_{\mu\nu} = -4g_{\mu\nu}$, and so the curvature scalar is just a constant. Given only these quantities the Schrödinger spacetimes would look just like an ordinary AdS_5 space, however the Weyl tensor for Schrödinger-Kaigorodov spaces remains nonvanishing, see also Appendix. Thus unlike pure AdS spaces which are conformally flat, the Schrödinger spaces will not be so. Also the isometries and local structure of spacetimes are completely different, see [34].

Especially for D6-branes from (32) we have

$$ds_{D6}^2 = R_6^2 z^3\bigg[4(-\frac{\beta^2}{z^4}(dx^+)^2 + \frac{-dx^+ dx^- + d\vec{x}_{(5)}^2}{z^2}) + \frac{dz^2}{z^2} + d\Omega_{(2)}^2 \bigg]$$
$$e^\phi = (2\pi)^{-4} g_{YM}^2 \big(\frac{z}{R_6^2}\big)^{\frac{3}{2}} \qquad (37)$$

where scaling invariance involves $z \to \xi z$, $x^- \to x^-$, $x^+ \to \xi^2 x^+$, $x^i \to \xi x^i$. Note x^- does not need to rescale in these 6-brane Lifshitz solutions, so they have a dynamical exponent $a = 2$. So only for D6 solutions, we can have x^- with arbitrary periodicity. Although these solutions have interesting value $a = 2$, but there is an overall Weyl scaling of metric and dilaton required.

3.1   A relationship of dynamical exponents

An intriguing but interesting aspect of our Lifshitz and Schrödinger p-brane solutions, in eqs. (14) and (32), is that they can be mapped into each other under the following exchange of the light-cone metric components

$$g_{++}^{(Lif)} \to -g_{--}^{(Sch)}, \quad g_{--}^{(Lif)} \to -g_{++}^{(Sch)}. \qquad (38)$$

While respective dynamical exponents get mapped as

$$a_{Lif} = 2 - a_{Sch} \qquad (39)$$

11

for all the Dp-branes. We have $a_{Lif} = 2(\frac{p-6}{p-5})$ while $a_{Sch} = \frac{2}{p-5}$, for all $0 < p \leq 6$ but $p \neq 5$. Especially, the D5-branes have $a_{Lif} = 1$. So the relation (39) is obeyed by all nonrelativistic Dp-branes presented here. This could be understood as follows. Under the Wick rotations (27), which takes AdS-BH into AdS-Bubble and vice versa, the lightcone coordinates map as

$$x^+ \rightarrow -ix^-, \quad x^- \rightarrow ix^+. \tag{40}$$

The above double Wick rotation [4] will exactly perform the operation given in (38), keeping g_{+-} fixed. Although related mathematically in this simple manner, the Lifshitz and Schrödinger p-Brane solutions represent physically distinct spacetimes, having different dynamical exponents. Thus it is not surprising that just as AdS-BH and Bubble solutions get mapped into eath other under double Wick rotations, the Lifshitz (14) and Schrödinger (32) spacetimes do as well map into each other. A comparative list of dynamical exponents is provided in the table (1).

p-brane	a_{Lif}	a_{Sch}	$(a_{Lif} + a_{Sch})$
1	5/2	- 1/2	2
2	8/3	-2/3	2
3	3	-1	2
4	4	-2	2
5	1	1	2
6	0	2	2

Table 1: Dynamical scaling exponents of the Lifshitz and the Schrödinger solutions

It is interesting to notice that the dynamical exponents of Lifshitz geometries are all positive. Especially, for 3-branes it is $a = 3$. We study it a bit further. Compactifying the Lifshitz solution (18) along x^- and S^5, the remaining 4-dimensional Lifshitz spacetime can be written in the Einstein frame as

$$ds^2_{Lif_4} \sim z[-\frac{(dx^+)^2}{\beta^2 z^6} + \frac{dx_1^2 + dx_2^2 + dz^2}{z^2}]. \tag{41}$$

Since the number of spatial dimensions is $d = 2$ here, the Lifshitz geometry (41) indeed represents a spacetime which has a hyperscaling exponent given as $\theta = 1$, and under the asymmetric scalings $x^+ \rightarrow \lambda^3 x^+$, $x^1 \rightarrow \lambda x^1$, $x^2 \rightarrow \lambda x^2$, $r \rightarrow \lambda r$ the metric (41) scales as

$$ds^2 \rightarrow \lambda^{\frac{2\theta}{d}} ds^2.$$

We also note that this $a = 3$ solution satisfies an important criterion $a \geq \frac{\theta}{d} + 1$ and also has $\theta = d - 1$. These conditions involve putting Null energy conditions on

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[4] We are thankful to the referee to have suggested this.

the energy-momentum tensor in the Lifshitz spacetime. These studies have been a subject of much attention recently in the literature, see [35, 36].

Similarly for other Lifshitz p-brane solutions in (14), after compactifications along x^- and S^{8-p}, we find that the hyperscaling dimensions are

$$\theta_{(p)} = \frac{p^2 - 6p + 7}{p - 5}, \quad p \neq 5. \tag{42}$$

and corresponding $(p+1)$-dimensional Lifshitz geometries can be written in Einstein frame as

$$ds^2_{Lif_{p+1}} \sim z^{\frac{2(p^2-6p+7)}{(p-1)(p-5)}} \left(-\frac{(dx^+)^2}{\beta^2 z^{\frac{4(p-6)}{(p-5)}}} + \frac{(d\vec{x}_{p-1})^2 + dz^2}{z^2} \right). \tag{43}$$

We find that physically interesting cases of $p = 2, 3, 4$ they all satisfy $a \geq \frac{\theta}{d} + 1$. These correspond to the cases where boundary CFTs have spatial dimensions $d = 1, 2, 3$ respectively. The Schrödinger geometries in the table are rather puzzling as all lower dimensional cases have negative dynamical exponents, except for $p = 5$ and $p = 6$. Also the spatial lightcone coordinate x^- is null there, so we cannot compactify along this direction. The problem we face here is similar to the case of Schrödinger solutions in [6]. It would be worth while to explore them further and we hope to return to them in near future.

4　Lifshitz singularities and thermodynamics

It has been recently shown that Lifshitz solutions have essential null curvature singularities [30]. It is due to fact that a test string become infinitely excited as it nears $r = 0$, even though the curvature scalar may be uniformally constant for the Lifshitz solutions (at least for the case of D3-branes). These instabilities would necessitate us to include higher (derivative) order corrections to these classical backgrounds. However, it would also be a safer approach to hide these essential Lifshitz singularities behind some event horizon. Correspondingly, the finite temperature Lifshitz solutions will be those which have finite horizon size r_0 and also finite boost. Let us write them down explicitly following from (7) (direction of the boost is taken to be along negative y direction)

$$
\begin{aligned}
ds^2 &= R_p^2 r^{\frac{p-3}{2}} \Bigg[r^{5-p} \left((1 + \frac{r_0^{7-p} v^2 \gamma^2}{r^{7-p}}) dy^2 - (1 - \frac{r_0^{7-p} \gamma^2}{r^{7-p}}) dt^2 - \frac{2v\gamma^2 r_0^{7-p}}{r^{7-p}} dt dy + d\vec{x}^2_{(p-1)} \right) \\
&\quad + \frac{dr^2}{fr^2} + d\Omega^2_{(8-p)} \Bigg] \\
e^\phi &= (2\pi)^{2-p} g_{YM}^2 (\frac{r^{7-p}}{R_p^4})^{\frac{p-3}{4}},
\end{aligned}
\tag{44}
$$

along with $(p+2)$-form flux. Introducing lightcone coordinates, $x^{\pm} = t \pm y$, the metric can be reexpressed as

$$
\begin{aligned}
ds^2 &= R_p^2 r^{\frac{p-3}{2}} \left[r^{5-p} \{ (\frac{r_0^{7-p}}{4r^{7-p}} \frac{(dx^+)^2}{\lambda^2} + \frac{r_0^{7-p}\lambda^2}{4r^{7-p}}(dx^-)^2 - \frac{1+f}{2} dx^+ dx^-) + d\vec{x}_{(p-1)}^2 \} \right. \\
&\quad \left. + \frac{dr^2}{fr^2} + d\Omega_{(8-p)}^2 \right] \\
&= R_p^2 r^{\frac{p-3}{2}} \left[\{ r^{5-p}(-f \frac{r^{7-p}}{W^2}(dx^+)^2 + d\vec{x}_{(p-1)}^2) + \frac{dr^2}{fr^2} \} + d\Omega_{(8-p)}^2 \right. \\
&\quad \left. + \frac{W^2}{4r^2}(dx^- - \frac{1+f}{r^{p-7}} \frac{dx^+}{W^2})^2 \right].
\end{aligned}
\tag{45}
$$

The parameters are related as $W^2 \equiv r_0^{7-p}\lambda^2$ and the function

$$
f = (1 - \frac{r_0^{7-p}}{r^{7-p}}),
$$

and $r = r_0$ is the horizon.[5] Note that we can rescale W out of these BH solutions, which is possible if we exploit the lightcone scaling $x^+ \to Wx^+$, $x^- \to W^{-1}x^-$. So we can set W to unity if we wish but it is an useful parameter in the following analysis. For very large W value these black holes in the intermediate range $r_0 < r < W^{\frac{2}{7-p}}$ will always behave as thermal Lifshitz like solutions. Let us call this region as the Lifshitz window region where parameter W provides the effective width of this window. While in the deep UV region, $r \gg W^{\frac{2}{7-p}}$ the solutions become asymptotically AdS, see the figure (1). Note that the size of Lifshitz window
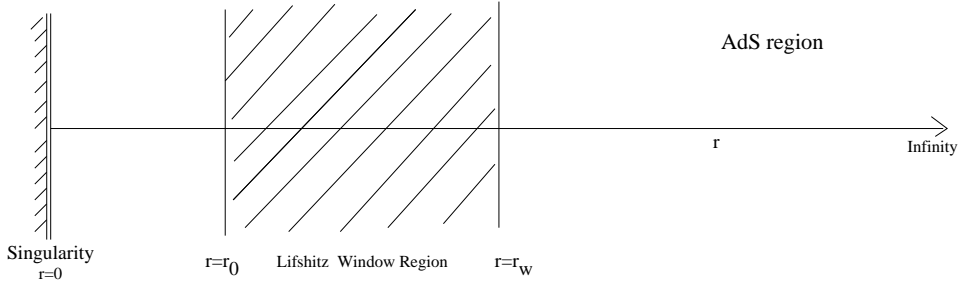

Figure 1: The Lifshitz window appears as the shaded region. It starts at r_0 and ends at $r_w \sim W^{\frac{2}{7-p}}$.

can be widened if we take λ (boost) sufficiently large. But if $\lambda = 1$ the Lifshitz region altogether disappears and we get ordinary AdS-BH solutions; see figure (2). We mentioned it earlier also that all our Lifshitz solutions become AdS spaces asymptotically. These Lifshitz BH solutions (45) with an intermediate Lifshitz region should present a good IR description (at finite temperature), where the black hole horizon

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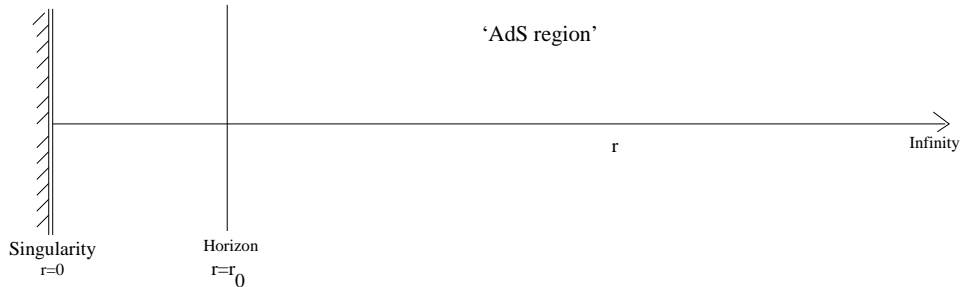[5] The boost parameter $\lambda \geq 1$ is written as $\lambda = \sqrt{\frac{1+v}{1-v}}$.

Figure 2: Depiction of an usual AdS black hole spacetime. There is no Lifshitz like region here.
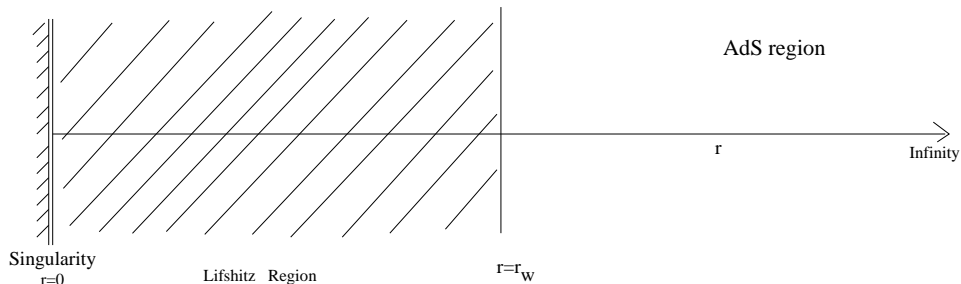

Figure 3: In zero temperature solutions the Lifshitz window (the shaded region) starts at $r = 0$ and ends at r_w.

size provides an effective IR (thermal) cut-off in the dual CFT. Of course, there is a black hole singularity but it will be hidden behind a horizon and cannot be seen by an asymptotic observer. So the issue of curvature singularity appears to be fine for black hole Lifshitz solutions (45). On the other hand our double limits in Eq.(9), however correspond to the widening of the Lifshitz window such that the left edge of the window moves to $r = 0$ while the right edge is held fixed at $r = r_w$. Consequently the limits give rise to zero temperature Lifshitz solutions given in equations (7) and (11). The zero temperature Lifshitz region is sketched in figure (3).

4.1   Double limits and thermodynamic duality

Having obtained the nonrelativistic geometries We shall need to study the effect of our double limits on the thermodynamic quantities. These finite temperature boundary field theories involve DLCQ description [6]. For our Lifshitz spacetimes, the compactification along x^- implies that there is a conserved charge (momentum) P_- which is quantized in units of $\frac{1}{r^-}$. The number (momentum) density depends upon the choice of two parameters, namely boost γ (or λ) and r_0. It would be worth while to know what happens to the number density, energy density $(-P_+)$ and other thermodynamical quantities; like temperature (T), entropy (S) and chemical potential (μ_N), relevant for the black-hole solutions, as we consider the double limits.

15

These thermodynamic expressions are considered here for the case of D3-branes for concreteness

$$\rho = \frac{N}{V_2} = \frac{r^-(-P_-)}{v_2} = \frac{L^3}{G_N^5}\frac{(r^-)^2\lambda^2 r_0^4}{8}$$

$$\mathcal{E} = \frac{H}{V_2} = \frac{(-P_+)}{v_2} = \frac{L^3}{G_N^5}\frac{r^- r_0^4}{16}$$

$$s = \frac{S}{V_2} = \frac{L^3}{4G_N^5}(2\pi r^-)\frac{\lambda r_0^3}{2}$$

$$T = \frac{r_0}{\pi\lambda}, \qquad \mu_N = \frac{1}{r^-\lambda^2} \tag{46}$$

where L represents the AdS_5 radius, G_N^5 is 5-dimensional Newton's constant and V_2 is some finite volume of $x_1 - x_2$ plane. These quantities eventually satisfy the first law of thermodynamics

$$\delta E(T, \mu_N) = T\delta S - \mu_N \delta N \tag{47}$$

We see that under the limits (9), the temperature of boundary $(2+1)$ dimensional nonrelativistic field theory effectively vanishes, so also the entropy and the chemical potential. Although these quantities are vanishing, worth noticing is their unique scaling behaviour as powers of vanishing horizon size r_0, observed initially in [21],

$$T \sim r_0^3 \sim 0, \quad \rho = \text{fixed}, \quad \mu_N \sim r_0^4 \sim 0, \quad s \sim r_0 \sim 0. \tag{48}$$

Especially, the temperature vanishes as cubic power of r_0, which is an indication of the fact that system behaves nonrelativistically having dynamical exponent $a = 3$ as it undergoes a condensation in the DLCQ theory. [21]. We can however reexpress (48) as

$$T \to 0, \quad \mu_N \to T^{\frac{4}{3}} \sim 0, \quad s \to T^{\frac{1}{3}} \sim 0, \quad \rho \sim \frac{T^4}{\mu_N^3} = \text{fixed} \tag{49}$$

These kind of strange behaviours, $s \sim T^{\frac{1}{3}}$, and specific heat $C = T\frac{\partial S}{\partial T} \sim T^{\frac{1}{3}}$, have recently been related to some non-Fermi liquids (strage metals) in 2d systems [35, 36]. Curiously though, we observe that the number density has to remain fixed in order to achieve this condensation. Obviously the energy density is also vanishing. We may also write down expressions for free energy density and the entropy density

$$F \sim -\mathcal{E} \sim -\frac{T^4}{\mu_N^2} \sim r_0^4 \sim 0, \quad s \sim \frac{T^3}{\mu_N^2} \sim r_0 \sim 0 \tag{50}$$

Let us compare above double limits i) $r_0 \to 0, \ \lambda = \infty$ with the ii) $r_0 \to 0, \ \lambda = \text{finite}$ limit. Under the latter kind of vanishing horizon (extremal) limit, the thermodynamical expressions behave as

$$T \sim r_0 \sim 0, \quad \mu_N \sim \text{fixed}, \quad \rho \sim r_0^4 \sim 0 \quad s \sim r_0^3 \sim 0 \tag{51}$$

16

and $\mathcal{E} \sim r_0^4 \sim 0$, as horizon size shrinks to zero. Particularly, the temperature vanishes as unit power of r_0 which is an indication of the fact that system behaves ordinarily relativistic, having dynamical exponent $a = 1$, as it undergoes a condensation. Juxtapose to the double limit case of (48), we find here that the chemical potential in (51) remains fixed while the number (momentum) density becomes vanishing, $O(r_0^4)$, in the relativistic situation.

From the above rather crude exercise and from the power law behaviour of the thermal quantities in (48) and (51), we would infer that two different limits describe two distinct types of condensation phenomenon; non-relativistic (Lifshitz) and the relativistic (Lorentzian). However, these two condensation points appear to be thermodynamically *dual* points. They could be Laplace transformed into each other,

$$T \leftrightarrow S, \quad \mu_N \leftrightarrow N, \quad \mathcal{E}(T, \mu_N) \leftrightarrow \tilde{\mathcal{E}}(S, N). \tag{52}$$

Since Laplace transformations of thermodynamic variables do map micro-canonical ensemble into a canonical/grand-canonical ensemble and vice versa, the two types of condensation ($T \to 0$) limits described above seem to represent two different physical ensembles near respective fixed points. We indeed observe that the condensation behaviour of the canonically conjugate variables, (T, S) and (μ_N, ρ) does seem to get mapped into each other.

5 Conclusion

We have extended our previous study of D3-branes where vanishing horizon double limits of 'boosted' $AdS_5 \times S^5$ black holes gave rise to Lifshitz type solutions with dynamical exponent $a = 3$ [21]. Taking the similar limits for all Dp-branes, we obtain a wide class of Lifshitz like solutions. However the asymmetric scaling of the coordinates has to be accompanied by a Weyl rescaling of the metric and dilaton field etc. This rescaling is identical to the standard Weyl scaling behaviour of Dp-branes AdS metrics [31]. Thus the Weyl rescaling of the string metric and e^ϕ is indicative of the fact that the boundary NRCFT is not conformal instead it has got running effective couplings. But what we find most interesting is that this RG flow of the NRCFT is still of the standard type. The new geometries so obtained describe nonrelativistic Lifshitz type dynamics with varied dynamical exponents, $a_{Lif} = 2\frac{p-6}{p-5}$. We have also studied similar double limits for boosted 'bubble' Dp-brane solutions. The resulting Galilean solutions instead are classified as Schrödinger type, with varied dynamical exponents as $a_{Sch} = \frac{2}{p-5}$. We did find an interesting map between these Lifshitz and Schrödinger type solutions, such that for a given p-brane case respective dynamical exponents get related as

$$a_{Lif} = 2 - a_{Sch} .$$

We also discussed thermodynamical aspects involving Lifshitz black hole solutions and the issue of null curvature singularities. We find that in order to study Lifshitz systems at finite temperature one can include black holes. In this way we can avoid seeing the essential Lifshitz singularities. Indepedently an observation regarding an intriguing thermodynamical duality is also reported.

As we were typesetting this work a few papers [37, 38] appeared in last couple of days which might see some overlap with us.

A Double limits of some flat space solutions and plane waves

Consider a Schwarzschild black hole solution in D spacetime dimensions which is delocalised (isometry) along one of the spatial coordinates

$$ds^2_{BH} = -f dt^2 + dy^2 + \frac{dr^2}{f} + r^2 d\Omega^2_{(D-3)}, \tag{53}$$

The function $f(r) = 1 - r_0^{D-4}/r^{D-4}$. This can also be called an uncharged black string and it has a asymptotic geometry $R^{D-1} \times R^1$.

Let us make the following boost along y,

$$dt \to \gamma dt + v\gamma dy, \quad dy \to \gamma dy + v\gamma dt \tag{54}$$

so we get a boosted BH geometry

$$ds^2 = (1 + \frac{r_0^{D-4} v^2 \gamma^2}{r^{D-4}}) dy^2 - (1 - \frac{r_0^{D-4} \gamma^2}{r^{D-4}}) dt^2 + \frac{2v\gamma^2 r_0^{D-4}}{r^{D-4}} dt dy + \frac{dr^2}{f} + r^2 d\Omega^2_{(D-3)} \tag{55}$$

We note that the periodicity of Euclidean time in the boosted geometry gets modified and it is now $\frac{4\pi r_0 \gamma}{D-4}$. Correspondigly horizon temperature after the boost is

$$T = \frac{D-4}{4\pi r_0 \gamma}. \tag{56}$$

Thus these BH solutions have two parameters r_0, γ.

The simultaneous $r_0 \to 0$, $\gamma \to \infty$ limit: We incorporate the double limit such that

$$r_0 \to 0, \quad \gamma \to \infty, \quad r_0^{D-4} \gamma^2 = \beta^2 = \text{fixed}. \tag{57}$$

In which case the BH temperature would behave as

$$T \sim O(r_0^{\frac{D-6}{2}}) \sim 0, \quad \text{for} \quad D > 6 \tag{58}$$

However such double limits exist only for $D > 6$. For $D = 5$ the temperature would blow up under the limit $r_0 \to 0$, so we do not include them here. Under the double limits we get following wave like solution (introducing light-cone coordinates)

$$ds^2 = \frac{\beta^2}{r^{D-4}}(dx^-)^2 - dx^+dx^- + dr^2 + r^2d\Omega^2_{(D-3)}, \quad \text{for} \quad D > 6 \tag{59}$$

We can notice that light-cone time is null, i.e. $g_{++} = 0$, in these wave solutions. These are the simplest type of plane-wave solutions in flat spacetime [32]. Actually the expression for g_{--} in Eq.(59) is nothing but a harmonic function over the transverse $(D-2)$-dimensional space.

Next we can consider a 'bubble' like spatial geometry (Euclidean Schwarzschild type geometry) in $D > 6$ spacetime dimensions

$$ds^2_{bubble} = -dt^2 + gdy^2 + \frac{dr^2}{g} + r^2d\Omega^2_{(D-3)}, \tag{60}$$

The function $g(r) = 1 - r_b^{D-4}/r^{D-4}$, with coordinate range $r_b \leq r \leq \infty$. The asymptotic spacetime geometry is $R^1 \times S^1 \times R^{(D-2)}$, having a y periodicity $y \sim y + \frac{4\pi r_b}{D-4}$. The asymptotic periodicity of y is chosen such that there is no singularity at $r = r_b$. Now we make a boost along y

$$ds^2 = -(1 + \frac{r_b^{D-4}v^2\gamma^2}{r^{D-4}})dt^2 + (1 - \frac{r_b^{D-4}\gamma^2}{r^{D-4}})dy^2 + \frac{2v\gamma^2 r_b^{D-4}}{r^{D-4}}dtdy + \frac{dr^2}{f} + r^2d\Omega^2_{(D-3)} \tag{61}$$

Note that the periodicity of y in the boosted geometry gets modified to $y \sim y + \frac{4\pi r_b\gamma}{D-4}$. Now consider the double limits

$$r_b \to 0, \quad \gamma \to \infty, \quad r_b^{D-4}\gamma^2 = \beta^2 = \text{fixed.} \tag{62}$$

Under this limit y circle essentially gets decompactified, if we keep $D > 6$. It gives another wave geometry

$$ds^2 = -\frac{\beta^2}{r^{D-4}}(dx^+)^2 - dx^+dx^- + dr^2 + r^2d\Omega^2_{(D-3)} \tag{63}$$

which is also a wave in empty space but has nontrivial g_{++} component. Thus our double limits generally produce wave like solutions.

B   Kaigorodov Spaces

Let us consider the following 5-dimensional metric which is a solution of the Einstein gravity with a negative cosmological constant,

$$ds^2_{Kaigorodov} = \pm\beta^2 z^2 (dx^{\mp})^2 + \frac{-dx^+ dx^- + dx_1^2 + dx_2^2}{z^2} + \frac{dz^2}{z^2} \tag{64}$$

where both plus and minus signs can be considered. In our conventions the solution with upper signs would classify as Lifshitz like, while with the lower signs we will identify it as Schrödinger spacetime. The Riemann tensor for this metric can be separated as

$$R_{\mu\nu\lambda\rho} = (g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + \delta_{\mu\nu\lambda\rho} \tag{65}$$

The expression $\delta_{\mu\nu\lambda\rho}$ collects deviations from pure AdS geometry (the β-dependent terms). For maximally symmetric Einstein spacetimes such as AdS, these extra terms are of course vanishing. But for above Kaigorodov-Lifshitz metrics (upper signs) the nonvanishing components are

$$\delta_{-1-1} = 2\beta^2, \quad \delta_{-2-2} = 2\beta^2, \quad \delta_{-z-z} = -4\beta^2, \tag{66}$$

While for Kaigorodov-Schroedinger metrics (lower signs) the nonvanishing components are

$$\delta_{+1+1} = -2\beta^2, \quad \delta_{+2+2} = -2\beta^2, \quad \delta_{+z+z} = 4\beta^2 \ . \tag{67}$$

Due to these contributions the corresponding Weyl tensor is nontrivial. As a consequence the Kaigorodov spacetimes are not conformally flat. But interestingly these corrections to the Riemann tensor do not contribute to the Ricci tensor. The Ricci tensor for Kaigorodov Lifshitz/Schroedinger spaces is still given as

$$R_{\mu\nu} = -4g_{\mu\nu}$$

just like for the pure AdS_5 spacetime. For the anti de Sitter spacetimes, which are also conformally flat, Weyl tensor is vanishing. More discussion about Kaigorodov spacetimes can be found in [34].

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